Mathematics learning in the PYP

Learning is improved if the whole-school community has a shared understanding of the long-term mathematics outcomes, and a commitment to achieving them. It has never been more important to teach mathematics well. Globalisation and the increasing use of technology have created changing demands for the application of mathematics in all aspects of our lives.

Learning is not simply about the transfer of knowledge from the teacher to the student. Rather, students need to construct their own mathematical understanding in their own way and at a pace that enables them to make sense of the mathematical situations and concepts they encounter. An approach to teaching with this in mind is based on a learning continuum. It recognises that not all students learn in the same way, through the same processes, or at the same rate.

A teacher collects data in order to identify where on those continuums a student is positioned to better design appropriate learning experiences. These experiences are designed to move the student from one development phase to the next. In this way, the learner is able to build on established skills and understanding, while being supported to meet appropriate challenges to extend their learning.
Acknowledging that learning mathematics is a developmental process, the ISUtrecht mathematics document presents a set of developmental continuums that are designed as tools to assist teachers in in monitoring students' development throughout their primary school experience with us.
The five language continuums in this document have been organized into various developmental phases with each phase building upon and complementing the previous one.

Some learners may enter the programme already demonstrating all of the outcomes in the first phase, while others may not demonstrate all of the outcomes in the higher levels when they transfer into secondary. However, the identification of possible learning outcomes across various phases allows teachers to plan learning experiences that are challenging, rigorous and relevant to each student's knowledge, understanding and skills, according to their previous experience, individual starting points and diverse ways of learning.

Role play phase

Usual grade levels: kindergarten through grade 2

In this phase, students develop awareness that some things are more and less likely to happen and begin to use some of the comparative language to describe likelihood.
They initially attend to overall appearance of size, recognising one thing as perceptually bigger than another and using comparative language in a fairly undifferentiated and absolute way rather than to describe comparative size.

As students move about their environment and explore the objects in it, they respond perceptually to spatial features, encoding shape and the location of objects they can see within a framework of landmarks.
Students reason about small amounts of physical materials, learning to distinguish small collections by size and recognising increases and decreases in them. They also learn to recognise and repeat the number words used and to distinguish number symbols from other symbols.

Some typical learning experiences from this phase include:

- looking at displays of familiar data, counting said data, and saying which is most or least
- sorting and arranging data into familiar groupings
- distinguishing tallness, heaviness, fatness and how much things hold and something as having more or less
- using opposites to describe things
- using informal language that indicates they are responding to shape
- carrying out shape matching tasks
- giving directions from one place to the next when retelling a journey or places in a story
- distinguishing spoken numbers from other spoken words and numerals from other written symbols
- seeing at a glance how many are in small collections and attaching the correct number names

Experimental phase

Usual grade levels: kindergarten through grade 3

Students in this phase draw on their experience to describe familiar things as more or less likely.
They match in a conscious way in order to decide which is bigger by familiar readily perceived and distinguished attributes such as length, mass, capacity and time.
Their exploration of objects and space through touch and sight gradually becomes more regulated as they attend to spatial features and construct mental and visual representations of shapes and arrangements in space.
Students use numbers as adjectives that describe actual quantities of physical materials. Through stories, games and everyday tasks, students use one-to-one relations to solve problems where they can directly carry out or imagine the actions suggested in the situation. They learn to fix small collections to make them match, 'deal out' collections or portions, and to respect most of the principles of counting.

Some typical learning experiences from this phase include:

- distinguishing impossible events from events that are possible but unlikely and between certain and uncertain events.
- using skip counting to say how many in a tally
- using counting to help construct their data display, e.g., construct a block graph by counting how many in each group, then counting how many squares to colour in
- understand the need for a baseline and space blocks regularly to allow comparisons to be made
- counting to compare the sizes of groups
- lining up the base of two sticks when comparing their lengths and fitting regions on top of each other to compare area
- counting units and call it 'measuring'
- describing figures and objects using terms that are evocative of shape
- learning the names of some shapes
- remember what some families of shapes look like and produce recognisable versions
- relating the position of objects to each other in familiar settings using terms such as 'behind', 'near'
- drawing or making simple 'route' maps and models that show a sense of spatial relationships and order
- recalling the sequence of number names at least into double digits
- knowing how to count a collection
- comparing two collections one to one and use this to decide which is bigger and how much bigger
- solving small number story problems which require them to add some, take some away, or combine two amounts by imagining or role playing the situation and counting the resulting quantity
- sharing by dealing out an equal number of items or portions to each recipient

Early phase

Usual grade levels: grade 1 through grade 5

Students connect the idea of likeliness to the frequency of an event. They come to understand that numbers can be used to describe chance and that counting frequencies can provide a measure of likelihood for future events.
They connect the two ideas of directly comparing the size of things and of deciding 'how many fit' and so come to an understanding that the count of actual or imagined repetitions of units gives an indication of size and enables two things to be compared without directly matching them.
Through their own physical and perceptual action on spatial configurations focused on interpreting, describing and representing the parts making the whole,

they make sense of the spatial relationships within figures, objects and arrangements and in the visual representations of them.
Students reason about numerical quantities and come to believe that if nothing is added to, or removed from, a collection or quantity, then the total amount must remain the same even if its arrangement or appearance is altered. They interpret small numbers as compositions of other numbers.

Some typical learning experiences from this phase include:

- constructing simple experiments and using counting to determine which event is more likely
- listing all possibilities for straightforward situations
- producing and reading pictographs or block graphs where each unit represents more than one piece of data
- producing simple two-way tables and Venn diagrams, partitioning totals between the cells or sections for straightforward data
- representing whole-number data in different ways
- using column and row headings to interpret what the numbers in simple two-way tables represent
- using units carefully to make as close a match as possible, avoiding gaps and overlaps
- seeing repeating one representation of the unit over and over as equivalent to filling or matching with multiple copies of it
- connecting the repetition of a 'unit' with the numbers on a whole-number calibrated scale
- making things to a specified length in uniform units
- using provided measurements to make a decision about comparative size
- counting units as a strategy to solve comparison
- adding measurements that they can readily think of in terms of repetitions of units
- using language such as 'flat', 'curved', 'side', 'round', 'face', 'edge', 'square', 'angle', 'base'
- comparing and contrasting geometric figures
- identifying the faces, edges and vertices of a geometric object and selecting component parts to make it in various forms
- matching the 2-dimensional shapes with the faces of standard 3-dimensional shapes
- selecting nets that have the right component parts to match a simple object
- paying attention to the shape and placement of component parts when they interpret and make drawings
- observing the component geometric parts within pictures and patterns and the movements needed to produce them
- rearranging and combining a few shape pieces to make another specified shape

- repeating multiple copies of a figure in a systematic way to create a pattern and recognizing this
- describing one thing being between others and put key features in order on a map
- Using counting as a strategy to solve problems
- using materials or visualisation to decompose small numbers into parts empirically
- selecting either counting on or counting back for subtraction problems depending on which strategy best matches the situation
- writing number sentences that match how they think about the story line for small number addition and subtraction problems

Transitioning phase

Usual grade levels: Grade 3 through secondary

Students begin to quantify the chance of events occurring using proportional measures to say "how likely" or "how much more likely." They realise they can produce ratios by comparing the total number of occurrences to the total number of trials or by comparing the number of desired outcomes to the number of possible outcomes.
They come to understand the unit as an amount and to see the process of matching a unit with an object as equivalent to subdividing the object into bits of the same size as the unit and counting the bits.
As they consciously compare and contrast spatial configurations, they form generalisations about relationships both within and between figures, objects, movements and arrangements. Through their own experimentation, they realise that when an object or arrangement is transformed, relationships between its component parts may be preserved or not, and they try to visualise 'what happens' when things are represented or moved.
Students come to see the significance of whole numbers having their own meaning independent of particular countable objects. They learn to use part-whole reasoning without needing to see or visualise physical collections. They trust, too, that appropriate partitioning of quantities must produce equal portions.

Some typical learning experiences from this phase include:

- drawing on personal experience to compare and order a variety of chance-related events and ordering them along a continuum from events that cannot happen to events that must happen
- drawing on numerical information to decide whether two simple events are or are not equally likely to occur
- ensuring that an experiment is fair
- listing all possibilities to work out numerical probabilities for one-stage actions

- using experimental results and data about past events to determine a range of possible outcomes and informally use relative frequency to estimate probabilities
- creating axes that show discrete or continuous quantities
- using simple proportional comparisons when interpreting data in tables and graphs
- composing 'part-units' into wholes
- partitioning a rectangle into appropriate squares and use the array structure to work out how many squares are in the rectangle
- interpreting the unnumbered graduations on a familiar whole-number scale
- working with provided measurement information alone to make things which meet measurement specifications
- giving a detailed list of properties in their descriptions of shapes
- selecting figures and objects based on geometric descriptions
- using mathematical conventions to represent objects in different types of drawings
- matching suitable nets to prisms and pyramids that are actually present by considering the shape and placement of the component parts
- producing their own nets for geometric shapes that they can see and handle
- describing characteristic features of mirror symmetry
- identifying the particular rotations, reflections and translations that relate the component parts of simple arrangements and patterns
- recognising and using a top view to represent familiar locations on plans using order and relative proximity among landmarks.
- comparing whole numbers using their knowledge of the patterns in the number sequence, and thinking of movements between numbers without actually or mentally representing the numbers as physical quantities
- explaining why any whole number can be rewritten as the addition of other numbers
- partitioning at least two- and three-digit numbers into standard component parts without reference to actual quantities
- counting up and down in tens from starting numbers like 23 or 79
- writing suitable number sentences for the range of addition and subtraction situations
- use the inverse relationship between addition and subtraction to make a direct calculation possible
- counting in multiplicative situations by representing one group and counting repetitions of that same group, simultaneously keeping track of the number of groups and the number in each group
- using successive splits to show that one half is equivalent to 2 parts in 4, 4 parts in 8, etc.
- partitioning a quantity into a number of equal portions to show unit fractions

Conventional phase

Usual grade levels: Grade 4 through secondary

Students come to trust measurement information, even when it is about things they cannot see or handle, and to understand measurement relationships, both those between attributes and those between units.

They develop coordinated mental representations of spatial configurations in relation to their component parts enabling them to mentally manipulate and transform figures, objects and arrangements. Through investigating properties of shapes and movements and inter-relationships between them, their use of visual images becomes constrained by their more abstract verbal knowledge of the properties. They also integrate distance and direction in their descriptions of paths and locations and can represent them on coordinate systems.

Students extend their additive ideas about whole numbers to include the coordination of two factors needed for multiplicative thinking. They learn to construct and coordinate groups of equal size, numbers of groups and a total amount. Students also learn to visualise multiplicative situations in terms of a quantity arranged in rows and columns (an array). They are able to relate different types of multiplication and division situations involving whole numbers. They also link the ideas of repeating equal groups, splitting a quantity into equal parts and fractions.

Some typical learning experiences from this phase include:

- using a range of information sources to put things in order from least likely to most likely
- using their understanding of equivalent fractions to judge equally likely events
- interpreting the 0 to 1 scale in general usage and understand why the probability that a toss of a fair die will produce 5 is one-sixth
- identifying all the outcomes for two- or three-stage situations
- simulating a situation where it would be difficult, costly, or inappropriate to generate real data, by designing simple experiments that replicate a significant aspect of the situation
- choosing from a wide range of measurement options when planning data investigations
- planning complex scales on axes to produce a wide range of graphs, including using class intervals, fractions, and percentages
- representing growth or change data over time by using a time scale, and approximate value within the intervals by joining the points when appropriate
- interpreting displays showing relational information between measurements or frequencies
- interpreting complex scales on graphs where not all scale markings are labelled
- subdividing units to make measurements more accurate

- choosing units that are sufficiently accurate to make the needed comparisons
- using their understanding of the multiplicative structure built into the metric system to move flexibly between related standard units
- noticing and rejecting unrealistic estimates and measurements
- using relationships between measurements to find measures indirectly
- using properties to convince themselves and others why a figure or object belongs to a class
- producing their own nets, considering in advance the level of precision needed to ensure the shape is correct in form and size, where tabs will be, and so on
- predict which face on nets will match which face on corresponding objects
- predicting the effect of particular movements (translations, rotations and reflections) on the orientation and position of figures and objects
- visualising an object or scene in different orientations, drawing other possible views of an object from information in 2-dimensional drawings.
- using their knowledge of place value to generate alternative partitions
- sustaining a correct whole number place-value interpretation in the face of conflicting information
- Using flexible mental partitioning of whole numbers
- visualising an array to see fractions and partitions
- visualising or drawing their own diagrams to compare fractions with the same denominator or simple equivalences
- splitting a whole into parts to understand mixed numbers
- relating fractions and division
- interpreting multiplication situations as 'times as much'
- selecting an appropriate multiplication or division operation on whole numbers including for problems that are not easily interpreted as 'lots of'